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Adaptive interconnected observer for sensorless induction motor

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An adaptive interconnected observer for induction motor (IM) drive without mechanical sensors (speed sensor and load torque sensor) is presented. The observer estimates the fluxes, the angular velocity, the load torque and the stator resistance even under or near unobservable conditions. Practical stability based on Lyapunov theory is proved to guarantee the strongly uniformly practical stability of the estimation error dynamics. A contribution of this article is the experimental validation of the observer on reference trajectories of a sensorless IM observer benchmark. The trajectories of this benchmark are chosen to test the motor near and under conditions of unobservability. Robustness with respect to parameters variations is proved and experimentally verified.

Keywords: induction motor; non-linear systems; adaptive observers; practical stability; low frequencies benchmark; sensorless

1. Introduction

Induction motors (IM) play an important part in many types of domestic and industrial processing machinery. The popularity of the IM is due to its ruggedness and operational reliability.

However, the IM presents a challenging control problem. This is mainly due to the following four factors:

- The IM is a complex highly coupled non-linear system.
- The rotor fluxes and speed are not usually measurable.
- Because of heating, the rotor and stator resistances considerably vary with a significant impact on the system dynamics.
- The load torque is generally unknown.

There are various methods to control motor torque and speed, varying in complexity, performance and cost. The vector techniques are used to adjust the instantaneous values of voltage and current, thus permitting high dynamic performance. Vector control can be implemented in many different ways. The well-known techniques are the field oriented control (FOC; Blaschke 1972) and the direct torque control (DTC; Takahashi and Noguchi 1986).

The FOC is based on resolving the instantaneous line input motor currents into two components, flux and torque, producing current components. FOC motor controllers are essentially current controller systems. In this way, it is expected that the motor will produce controllable torque similar to the separately excited DC drive. In separately excited DC drives, the torque produced is a function of the magnetic field flux linkage and armature current component and how the armature windings are connected.

For direct FOC and variable structure control (VSC) of IM drive, for example, the speed knowledge is crucial, and generally sensors are used to measure it. However, in the high-power range many sensors are used and their maintenance is difficult. Vibrations produced by the high-power motor damage the encoder coupling and the speed measure quality. Consequently, during the last decade, there has been a considerable interest to develop IM drive without mechanical sensors (sensorless). A major difficulty is the estimation of the state variables at low frequencies. Another difficulty is to ensure the robustness against parameter variations. For example, the most critical parameter affecting performance at low speed is the stator resistance (Holtz 2002; Montanari and Tilli 2006). In these papers, it is shown that relevant uncertainties in stator resistance value introduce speed and flux estimation error, leading to uncorrect speed, and flux tracking in control loop.

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In the literature, several approaches have been proposed to estimate the rotor velocity, rotor fluxes and load torque from the stator current and voltage measurements such as:

- Flux estimation using rather simple models with explicit compensation of non-linearities and disturbances (Holtz and Quan 2002).
- State estimation based on high-frequency signal injection and/or saliency-induced effects on the stator voltage (Jansen and Lorenz 1995; Holtz 2000).
- Estimation using adaptive and/or robust observers based on fundamental excitation and advanced models (Kubota, Matsuse, and Nakano 1993; Montanari, Peresada, and Tilli 2003). In Kubota et al. (1993), the estimation is obtained by means of the cross product of current error vector and the observed flux vector.
- Sliding mode techniques to design observer (Tursini, Petrella, and Parasiliti 2000; Barambones and Garrido 2004).
- Cascade observer, schemes based on cascade structure of the electrical and mechanical parts of the IM (Schreier, Leon, Glumineau, and Boisliveau 2001; Ghanes, Leon, and Glumineau 2005).
- Interconnected high-gain observers in Besançon and Hammouri (1998), Ghanes, Huerta, Leon, and Glumineau (2004) and Ghanes, Leon, and Glumineau (2006).

In Ghanes et al. (2004), a high-gain observer connected to an estimator has been proposed to reconstruct flux and speed. Furthermore, in order to improve this observer design, in Ghanes et al. 2006) two interconnected observers have been used to estimate the unmeasurable variables.

In Schreier et al. (2001) and Ghanes et al. (2006), the main purpose of these observers is to estimate the state variables and the load torque by taking into account the IM observability problem at low frequency.

Canudas, Youssef, Barbot, Martin, and Malrait (2000) and Ibarra, Moreno, and Espinosa (2004) demonstrate that the main conditions to lose the observability of IM are: the excitation voltages frequency is zero and the rotor speed is constant. In Ghanes et al. (2006), to avoid the bad behaviour of observer at low frequencies (when the motor is near or under conditions of unobservability, the authors proposed the switching method based on the evaluation of the determinant of observability matrix, such that when the system is in under conditions of unobservability, the gain of the observer is turned off. Then the observer may be seen as an estimator with a limited robustness with respect to IM parameters variations. Theory of stability in the sense of Lyapunov is widely used to investigate properties of non-linear systems in real applications. It is obvious that for asymptotic stability an important feature is to know the size of the region where we can judge whether or not a given system is stable.

As for the IM, the observability properties are lost at very low speed, it is well known that it is impossible to reconstruct the state that asymptotically converges to indistinguishable trajectories (Ibarra et al. 2004). However, under these trajectories, it is possible to design an observer whose performances are acceptable even if the asymptotic stability cannot be guaranteed. Thus for these practical considerations, it is clear that we need a notion of stability that is more suitable than asymptotic stability. Such a notion is the *practical stability* (Laskhmikanthan, Leela, and Martynyuk 1990).

1.1 *Motivation*

In order to obtain a better performance by using a controller, it is necessary to know all parameters and measure all variables of a system. However, it is not possible to have the information of all variables of the system because this requires to implement a lot of sensors, which are so expensive or are not physical to implement in the system, or they do not exist. On the other hand, usually not all parameters are known exactly or they change on the time. For these reasons, it is necessary to estimate the non-measurable variables and identify the unknown parameters, in order to implement a control law.

The main purpose of this article is to improve the observer design proposed in Traoré, DeLeon, Glumineau, and Loron (2006) by adding an estimation of stator resistance which avoids the above-mentioned phenomenon caused by effect of stator resistance value error. The new observer is called 'adaptive interconnected observer', which preserves the basic properties of the proposed interconnected observer.

Analysing the effects of a wrong stator resistance value on the algorithm of Traoré et al. (2006), it can be noted that a steady-state estimation error arises for load torque, motor speed and flux when the IM works at very low speed (near and under unobservable conditions). Consequently, the main idea is to design an observer that can estimate the angular speed, the flux, the load torque and additionally the stator resistance (critical parameter at very low speed). A suitable and simple interconnected form of IM has been proposed including a new stator resistance estimation mechanism in the observer design when comparing to the previous works of Ghanes et al. (2006) and Traoré et al. (2006).

1.2 *Contribution*

In this article, we propose an interconnected adaptive observer to simultaneously estimate the flux and speed and identify the load torque and stator resistance for a sensorless IM.

The goal of the proposed interconnected adaptive observer is to combine the knowledge of the inputs (stator voltages) and the measured outputs (currents) of the sensorless IM to solve the on-line estimation of the non-measurable states (the flux and the speed), a physical parameter (stator resistance) and the load torque. Furthermore, this observer improves the observer proposed in Traoré et al. (2006) and the robustness with respect to stator resistance variation. Sufficient conditions are given to prove the practical stability of this observer. These conditions are in terms of the boundedness of parameters variations. Finally, experimental results are given, on a significant benchmark described in Ghanes et al. (2006) to illustrate the performance of the observer.

The main advantages of our adaptive observer are (i) its stability can be proved under reasonable conditions, (ii) it does not need any dynamical model of the parameters variations and (iii) the stability of the observer under or near the unobservable conditions is guaranteed, i.e. sufficient conditions are given in order to guarantee the practical stability of the sensorless IM.

1.3 *Paper structure*

This article is organised as follows. Section 2 is devoted to the model of IM. The adaptive interconnected observer design is introduced in Section 3. In Section 4, an analysis of the observer convergence using the practical stability theory is given. Experimental results are given and discussed in Section 5. Finally, some conclusions are drawn.

2. Introduction motor model

The IM model, described in this article, is based on the motor equations in a rotating **d** and **q**-axes.[1] The IM dynamics behaviour is described by

$$\begin{bmatrix} \dot{i}_{sd} \\ \dot{i}_{sq} \\ \dot{\phi}_{rd} \\ \dot{\phi}_{rq} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} ba\phi_{rd} + bp\Omega\phi_{rq} - \gamma i_{sd} + \omega_s i_{sq} \\ ba\phi_{rq} - bp\Omega\phi_{rd} - \gamma i_{sq} - \omega_s i_{sd} \\ -a\phi_{rd} + (\omega_s - p\Omega)\phi_{rq} + aM_{sr} i_{sd} \\ -a\phi_{rq} - (\omega_s - p\Omega)\phi_{rd} + aM_{sr} i_{sq} \\ m(\phi_{rd} i_{sq} - \phi_{rq} i_{sd}) - c\Omega - \frac{1}{J} T_l \end{bmatrix} + \begin{bmatrix} m_1 & 0 \\ 0 & m_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} \quad (1)$$

where i_{sd}, i_{sq}, ϕ_{rd}, ϕ_{rq}, u_{sd}, u_{sq}, Ω, T_l and ω_s are the stator currents, the rotor fluxes, the stator voltage inputs, the angular speed, the load torque and the stator frequency, respectively. The subscripts s and r refer to the stator and rotor. The parameters a, b, c, γ, σ, m and m_1 are defined as $a = R_r/L_r$, $b = M_{sr}/\sigma L_s L_r$, $c = f_v/J$, $\gamma = \frac{L_r^2 R_s + M_{sr}^2 R_r}{\sigma L_s L_r^2}$, $\sigma = 1 - (M_{sr}^2/L_s L_r)$, $m = pM_{sr}/JL_r$, $m_1 = 1/\sigma L_s$. R_s and R_r are the resistances. L_s and L_r are the self-inductances, M_{sr} is the mutual inductance between the stator and rotor windings. p is the number of pole-pair. J is the inertia of the system (motor and load) and f_v is the viscous damping coefficient. Let $\gamma_1 = \gamma - m_1 R_s$ which is needed in the sequel of the study.

$\rho(\dot{\rho} = \omega_s)$ and ω_s are respectively the angular position and speed of the **dq**-frame with respect to a fixed stator reference frame $\alpha\beta$, where the physical variables are defined. Transformed variables in (1) are given by

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = P(\rho) \begin{bmatrix} x_d \\ x_q \end{bmatrix}, \quad \begin{bmatrix} x_d \\ x_q \end{bmatrix} = P(-\rho) \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

where $P(\rho) = \begin{bmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{bmatrix}$.

Remark 2.1: ρ is the relative angle between stator and rotor, that is needed to do a transformation from natural phase currents to **dq**-reference frame. ρ is derived from ω_s ($\dot{\rho} = \omega_s$). The stator frequency (ω_s) can be provided by considering the two cases:

- the observer is used with a control law (e.g. scalar control) which can give the stator frequency,
- the observer is used with a control law that cannot give the stator frequency, or the observer is used without controller (fault detection, diagnosis, etc.). In this case, the stator frequency must be estimated. □

3. Adaptive interconnected observers design

It is clear that in the literature there are several contributions in adaptive observer design. For instance, in Montanari et al. (2003) an adaptive observer is proposed to estimate the non-measurable variables and to identify the rotor resistance. Sufficient conditions are given to prove the convergence of this observer.

Combining sliding mode techniques and adaptation methods, an adaptive sliding mode observer scheme has been proposed in Furuhashi (1990).

Furthermore, in Marino (1990) sufficient and necessary conditions are given for a non-linear system to be transformable by state-space change of coordinates into a special adaptive observer form. Other contributions propose an adaptive observer to estimate the speed and flux and assuming either the load torque or the stator resistance is known.

This section displays the design of an adaptive interconnected observer (Besançon and Hammouri 1998; Besançon, Leon, and Huerta 2006) for the sensorless IM. It is assumed that load torque and stator resistance are slowly varying with respect to electric and mechanic variables. Then the dynamic behaviour of these two variables can be read as

$$\dot{T}_l = 0 \quad \dot{R}_s = 0. \tag{2}$$

Remark 3.1: Equation (2) means that the load torque and stator resistance values are assumed to be approximate by piecewise constant function. Only the bound of the load torque is assumed to be known. Furthermore, it is clear that the stator resistance slowly changes with the temperature. However, using step constant functions this variation can be approximated and the proposed approach works. Other approaches can be used, for instance singular perturbation methodology; however, the dynamics of the IM is fast with respect to the variations of the stator resistance that it could be considered constant. □

Thus, the extended IM model (1) and (2) may be seen as the interconnection between two subsystems

$$\Sigma_1 \begin{cases} \dot{X}_1 = A_1(X_2, y)X_1 + g_1(u, y, X_2, X_1) + \Phi T_l \\ y_1 = C_1 X_1 \end{cases} \tag{3}$$

and

$$\Sigma_2 \begin{cases} \dot{X}_2 = A_2(X_1)X_2 + g_2(u, y, X_1, X_2) \\ y_2 = C_2 X_2 \end{cases} \tag{4}$$

with

$$A_1(\cdot) = \begin{bmatrix} 0 & bp\phi_{rq} & -m_1 i_{sd} \\ -m\phi_{rq} & -c & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A_2(\cdot) = \begin{bmatrix} -\gamma_1 & -bp\Omega & ab \\ 0 & -a & -p\Omega \\ 0 & p\Omega & -a \end{bmatrix},$$

$$g_1(\cdot) = \begin{bmatrix} -\gamma_1 i_{sd} + ab\phi_{rd} + m_1 u_{sd} + \omega_s i_{sq} \\ m\phi_{rd} i_{sq} \\ 0 \end{bmatrix},$$

$$g_2(\cdot) = \begin{bmatrix} -m_1 R_s i_{sq} - \omega_s i_{sd} + m_1 u_{sq} \\ \omega_s \phi_{rq} + aM_{sr} i_{sd} \\ -\omega_s \phi_{rd} + aM_{sr} i_{sq} \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0 \\ -\frac{1}{J} \\ 0 \end{bmatrix},$$

$$C_1 = C_2 = [\ 1\ 0\ 0\ ].$$

$X_1 = [i_{sd}\ \Omega\ R_s]^T$ and $X_2 = [i_{sq}\ \phi_{rd}\ \phi_{rq}]^T$ are respectively the state vectors of (3) and (4), $u = [u_{sd}\ u_{sq}]^T$ is the input, and $y = [i_{sd}\ i_{sq}]^T$ is the output of the IM model. Furthermore, the IM physical operation domain $\mathcal{D}$ is defined by the set of values

$$\mathcal{D} = \{X \in R^7 \mid |\phi_{rd}| \le \Phi_d^{\max}, |\phi_{rq}| \le \Phi_q^{\max}, \\ i_{sd}| \le I_d^{\max}, |i_{sq}| \le I_q^{\max}, |\Omega| \le \Omega^{\max}, \\ |T_l| \le T_l^{\max}, |R_s| \le R_s^{\max}\}$$

with $X = [\phi_{rd}\ \ \phi_{rq}\ \ i_{sd}\ \ i_{sq}\ \ \Omega\ \ T_l\ \ R_s]^T$ and $\Phi_d^{\max}$, $\Phi_q^{\max}$, $I_d^{\max}$, $I_q^{\max}$, $\Omega^{\max}$, $T_l^{\max}$, $R_s^{\max}$ the actual maximum values for fluxes, currents, speed, torque load and stator resistance, respectively.

Remark 3.2: The choice of the variables of each subsystem has been considered in order to separate the mechanical variables (Ω, T_l, R_s) from the magnetic variables (ϕ_{rd}, ϕ_{rq}). It is clear that other choice could be considered in order to represent these subsystems, provided an observer could be designed. □

The adaptive interconnected observer, developed in the sequel for the sensorless IM, is based on the interconnection between several observers satisfying some required properties, in particular the property of input persistence (Besançon and Hammouri 1996). As defined in this latter reference, the input persistence is related to the observability properties of system (3) and (4).

In order to design an observer for system (3) and (4), a separate synthesis of the observer for each subsystem is required.

Remark 3.3:

(1) X_2 and X_1 are respectively considered as inputs for subsystems (Σ_1) and (Σ_2). From Besançon and Hammouri (1996), solutions of $\dot{S}_1$ and $\dot{S}_2$ (used below for the observer design) are symmetric positive definite matrices.

(2) When the IM remains in the observable area, X_2 and X_1 satisfy the regularly persistence

condition: then, asymptotic stability of the observer is guaranteed.

(3) When the IM remains in the unobservable area, X_2 and X_1 do not satisfy the regularly persistence condition. Then, asymptotic stability of the observer is not guaranteed. This problem is solved, by using the practical stability introduced in Section 4. □

Remark 3.4: From (3) and (4), it is clear that $A_1(X_2, y)$ is globally Lipschitz w.r.t. X_2, $A_2(X_1)$ is globally Lipschitz w.r.t. X_1. $g_1(u, y, X_2, X_1)$ is globally Lipschitz w.r.t. X_2, X_1 and uniformly w.r.t. (u, y) and that $g_2(u, y, X_2, X_1)$ is globally Lipschitz w.r.t. X_2, X_1 and uniformly w.r.t. (u, y). □

Then, adaptive interconnected observers for subsystems (3) and (4) are given by

$$O_1: \begin{cases} \dot{Z}_1 = A_1(Z_2, y)Z_1 + g_1(u, y, Z_2, Z_1) + \Phi\hat{T}_l \\ \qquad + (\varpi\Lambda S_3^{-1}\Lambda^T C_1^T + \Gamma S_1^{-1} C_1^T)(y_1 - \hat{y}_1) \\ \qquad + KC_2^T(y_2 - \hat{y}_2) \\ \dot{\hat{T}}_l = \varpi S_3^{-1}\Lambda^T C_1^T(y_1 - \hat{y}_1) \\ \qquad + B_1(Z_2)(y_2 - \hat{y}_2) + B_2(Z_2)(y_1 - \hat{y}_1) \\ \dot{S}_1 = -\theta_1 S_1 - A_1^T(Z_2, y)S_1 - S_1 A_1(Z_2, y) + C_1^T C_1 \\ \dot{S}_3 = -\theta_3 S_3 + \Lambda^T C_1^T C_1 \Lambda \\ \dot{\Lambda} = (A_1(Z_2, y) - \Gamma S_1^{-1} C_1^T C_1)\Lambda + \Phi \\ \hat{y}_1 = C_1 Z_1 \end{cases} \tag{5}$$

$$O_2: \begin{cases} \dot{Z}_2 = A_2(Z_1)Z_2 + g_2(u, y, Z_1, Z_2) + S_2^{-1}C_2^T(y_2 - \hat{y}_2) \\ \dot{S}_2 = -\theta_2 S_2 - A_2^T(Z_1)S_2 - S_2 A_2(Z_1) + C_2^T C_2 \\ \hat{y}_2 = C_2 Z_2 \end{cases} \tag{6}$$

with $Z_1 = [\hat{i}_{sd}\ \hat{\Omega}\ \hat{R}_s]^T$ and $Z_2 = [\hat{i}_{sq}\ \hat{\phi}_{rd}\ \hat{\phi}_{rq}]^T$ are the estimated state variables respectively of X_1 and X_2. θ_1, θ_2, θ_3 are positive constants, S_1 and S_2 are symmetric positive definite matrices (Besançon and Hammouri 1996), $S_3(0) > 0$, $B_1(Z_2) = km\hat{\phi}_{rd}$, $B_2(Z_2) = -km\hat{\phi}_{rq}$,

$$K = \begin{bmatrix} -k_{c1} & 0 & 0 \\ -k_{c2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{bmatrix}$$

with k, k_{c1}, k_{c2}, α and ϖ are positive constants.

We can see that the first observer (5) is constituted of two parts: one part to estimate the state ($i_{sd}\ \Omega\ R_s$) and the second part to estimate the load torque (T_l), by using the stator currents i_{sd} and i_{sq}. This second part depends on a differential equation representing a dynamical system described in terms of Λ (the state of this system) and Φ (the input matrix).

Furthermore, $(\varpi\Lambda S_3^{-1}\Lambda^T C_1^T + \Gamma S_1^{-1} C_1^T)$ and KC_2^T are the gains of observer (5) and $S_2^{-1}C_2^T$ is the gain of observer (6).

The gain of the observer (5) is split into two terms. The first one, $(\Gamma S_1^{-1} C_1^T)$, is associated to the state estimation and depends on the solution of a Ricatti equation. The second one $(\varpi\Lambda S_3^{-1}\Lambda^T C_1^T)$ is related to the identification parameter and depends on the solution of a differential equation. The solutions of these equations are dependent of the regularly persistence (richness of the signal) with respect to state and the parameter, respectively.

Remark 3.5: In Equation (5) the term $(B_1(Z_2)(y_2 - \hat{y}_2) + B_2(Z_2)(y_1 - \hat{y}_1))$ can be expressed as follows:

$$\begin{aligned} B_1(Z_2)(y_2 - \hat{y}_2) + B_2(Z_2)(y_1 - \hat{y}_1) \\ \equiv k[m(\hat{\phi}_{rd} i_{sq} - \hat{\phi}_{rq} i_{sd}) - m(\hat{\phi}_{rd}\hat{i}_{sq} - \hat{\phi}_{rq}\hat{i}_{sd})] \\ \equiv k(T_e - \tilde{T}_e) \end{aligned}$$

where T_e and $\tilde{T}_e$ are the 'measured' and 'estimated' electromagnetic torques, respectively. □

Lemma 3.6 (Besançon and Hammouri 1996)**:** Assume that v is a regularly persistent input for state affine system (3) and (4), and consider the following Lyapunov differential equation:

$$\dot{S}(t) = -\theta S(t) - A^T(v(t))S(t) - S(t)A(v(t)) + C^T C$$

with $S(0) > 0$, then

$$\exists\theta_0 > 0, \quad \forall\theta \geq \theta_0, \quad \exists\bar{\alpha} > 0, \quad \bar{\beta} > 0, \quad t_0 > 0:$$

$$\forall t \geq t_0, \quad \bar{\alpha} I \leq S(t) \leq \bar{\beta} I,$$

where I is the identity matrix (see the proof in Besançon and Hammouri (1996)). □

It is clear that $v = (u, X_2)$ and $S(t) = S_1$ for subsystem (3), and for subsystem (4) one has $v = (u, X_1)$ and $S(t) = S_2$.

It is worth mentioning that the conditions of observability loss have been stated in Ibarra et al. (2004), where the IM is unobservable under some inputs (the rotor speed constant and stator frequency set to zero simultaneously). In the IM observability area, the inputs $v = (u, X_2)$ and $v = (u, X_1)$, for subsystem (3) and (4), respectively, are regularly persistent and the convergence of the observer can be assured. However, in the unobservable region IM (under the conditions of speed constant and stator frequency set to zero), such inputs are 'bad input' and the observer convergence is not guaranteed. The use of practical stability properties can solve this problem.

4. Stability analysis of observer under uncertain parameters

Under indistinguishable trajectories (unobservable area) (Ibarra et al. 2004), the asymptotic convergence of any observer cannot always be guaranteed because the observability properties are lost on these trajectories. Then, in such cases, it is necessary to analyse the stability of the observer and the closed loop system. The practical stability notion (Laskhmikanthan et al. 1990) allows to establish that dynamics of the estimation error converge in a ball B_r of radius r ($x \in B_r \Rightarrow \|x\| \leq r$). If $r \to 0$ at $t \to \infty$, then the classical asymptotic stability is obtained.

4.1 *Preliminary results*

This part is essentially devoted to introduce some concepts and results of practical stability properties using Lyapunov-like functions and the theory of differential inequalities (Laskhmikanthan et al. 1990). Define the following class of function: $\mathbf{W} = \{d_1 \in C[\mathbb{R}_+, \mathbb{R}_+]$: $d_1(l)$ is strictly increasing in l and $d_1(l) \to \infty$ as $l \to \infty\}$. Let $B_r = \{e \in \mathbb{R}^n : \|e\| \leq r\}$ where $e = (\epsilon_i, i = 1, \ldots, n)^T$.

Consider the dynamical system

$$\dot{e} = f(t, e), \quad e(t_0) = e_0, \quad t_0 \geq 0. \tag{7}$$

Then system (7) is said to be:

(PS1) Uniformly practical stable if, given $(\hbar_1, \hbar_2)$ with $0 < \hbar_1 < \hbar_2$, we have

$$\|e_0\| \leq \hbar_1 \Rightarrow \|e(t)\| \leq \hbar_2, \quad \forall t \geq t_0, \quad \forall t_0 \in \mathbb{R}_+.$$

(PS2) Uniformly practical quasi-stable if, given $\hbar_1 > 0$, $\Im > 0$, $T > 0$ and $\forall t_0 \in \mathbb{R}_+$, we have

$$\|e_0\| \leq \hbar_1 \Rightarrow \|e(t)\| \leq \Im, \quad t \geq t_0 + T.$$

(PS3) Strongly uniformly practical stable, if (PS1) and (PS2) hold together.

Theorem 4.1 (Laskhmikanthan et al. 1990)**:** *Assume that*:

(i) $\hbar_1$, $\hbar_2$ are given such that $0 < \hbar_1 < \hbar_2$;
(ii) $V \in C[\mathbb{R}_+ \times \mathbb{R}^n, \mathbb{R}_+]$ and $V(t, e)$ is locally Lipschitzian in e;
(iii) for $(t, e) \in \mathbb{R}_+ \times B_{\hbar_2}$, $d_1(\|e\|) \leq V(t, e) \leq d_2(\|e\|)$ and

$$\dot{V}(t, e) \leq \wp(t, V(t, e)) \tag{8}$$

where d_1, $d_2 \in \mathbf{W}$ and $\wp \in C[\mathbb{R}_+^2, \mathbb{R}]$;
(iv) $d_2(\hbar_1) < d_1(\hbar_2)$ holds.

Then, the practical stability properties of:

$$\dot{l} = \wp(t, l), \quad l(t_0) = l_0 \geq 0, \tag{9}$$

imply the corresponding practical stability properties of system (7). □

Corollary 4.2 (Laskhmikanthan et al. 1990)**:** In Theorem 4.1, $\wp(t, l) = -\alpha_1 l + \alpha_2$, with α_1 and $\alpha_2 > 0$, implies strong uniform practical stability of system (7). □

For the proofs of Theorem 4.1 and Corollary 4.2, we refer to Laskhmikanthan et al. (1990).

4.2 *Stability analysis*

Consider that the IM parameters are uncertain bounded with well-known nominal values. Then, Equations (3) and (4) can be rewritten as

$$\begin{aligned} \dot{X}_1 &= A_1(X_2, y)X_1 + g_1(u, y, X_2, X_1) + \Phi T_l \\ &\quad + \Delta A_1(X_2, y) + \Delta g_1(u, y, X_2, X_1) \\ y_1 &= C_1 X_1 \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{X}_2 &= A_2(X_1)X_2 + g_2(u, y, X_1, X_2) \\ &\quad + \Delta A_2(X_1) + \Delta g_2(u, y, X_1, X_2) \\ y_2 &= C_2 X_2 \end{aligned} \tag{11}$$

with $\Delta A_1(X_2, y)$, $\Delta A_2(X_1)$, $\Delta g_1(u, y, X_2, X_1)$ and $\Delta g_2(u, y, X_1, X_2)$ are the uncertain terms of $A_1(X_2, y)$, $A_2(X_1)$, $g_1(u, y, X_2, X_1)$, $g_2(u, y, X_1, X_2)$, respectively. Note that R_r^{id}, R_s^{id}, M_{sr}^{id}, J^{id}, L_s^{id} and L_r^{id} are the identified parameters. Because of experimental conditions (e.g. temperature variation, imprecision of identification method), the identified parameters are not exactly the real parameters of the IM. Then, one has $b = b^{id} + \Delta b$, $a = a^{id} + \Delta a$, $c = c^{id} + \Delta c$, $m = m^{id} + \Delta m$, $m_1 = m_1^{id} + \Delta m_1$, $\gamma_1 = \gamma_1^{id} + \Delta\gamma_1$, with b^{id}, a^{id}, c^{id}, m^{id}, m_1^{id}, γ_1^{id}, Δb, Δa, Δc, Δm, Δm_1 and $\Delta\gamma_1$ are the identified values and uncertain values for b, a, c, m, m_1 and γ_1, respectively. It follows that the uncertain terms are represented as

$$\Delta A_1(\cdot) = \begin{bmatrix} 0 & \Delta b \cdot p\phi_{rq} & -\Delta m_1 \cdot i_{sd} \\ -\Delta m \cdot \phi_{rq} & -\Delta c & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Delta g_1(\cdot) = \begin{bmatrix} -\Delta\gamma_1 \cdot i_{sd} + \Delta ab \cdot \phi_{rd} + \Delta m_1 \cdot u_{sd} + \omega_s i_{sq} \\ \Delta m \cdot \phi_{rd} i_{sq} \\ 0 \end{bmatrix}.$$

$\Delta A_2(X_1)$, $\Delta g_2(u, y, X_2, X_1)$ can be written following a similar way.

By considering the IM physical operation domain $\mathcal{D}$, then there exist positive constants $\rho_i > 0$, for $i = 1, \ldots, 4$; such that

$$\|\Delta A_1(X_2, y)\| \le \rho_1, \quad \|\Delta A_2(X_1)\| \le \rho_2,$$
$$\|\Delta g_1(u, y, X_2, X_1)\| \le \rho_3, \quad \|\Delta g_2(u, y, X_1, X_2)\| \le \rho_4.$$

The parameters ρ_i, $i = 1, \ldots, 4$ are positive constants determined from the maximal values of $\Delta A_1(\cdot)$, $\Delta A_2(\cdot)$, $\Delta g_1(\cdot)$ and $\Delta g_2(\cdot)$ in the physical domain $\mathcal{D}$.

Let the estimation errors define as

$$\epsilon_1' = X_1 - Z_1, \quad \epsilon_2 = X_2 - Z_2, \quad \epsilon_3 = T_l - \hat{T}_l. \tag{12}$$

From Equations (5) and (6) and (10) and (11), one gets

$$\begin{aligned}\dot{\epsilon}_1' = {} & [A_1(Z_2, y) - \varpi \Lambda S_3^{-1} \Lambda^T C_1^T C_1 - \Gamma S_1^{-1} C_1^T C_1]\epsilon_1' \\ & + \Phi \epsilon_3 - K C_2^T C_2 \epsilon_2 + [A_1(X_2, y) + \Delta A_1(X_2, y) \\ & - A_1(Z_2, y)]X_1 + g_1(u, y, X_2, X_1) \\ & + \Delta g_1(u, y, X_2, X_1) - g_1(u, y, Z_2, Z_1)\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{\epsilon}_2 = {} & [A_2(Z_1) - S_2^{-1} C_2^T C_2]\epsilon_2 + [A_2(X_1, y) \\ & + \Delta A_2(X_1, y) - A_2(Z_1, y)]X_2 + g_2(u, y, X_1, X_2) \\ & + \Delta g_2(u, y, X_1, X_2) - g_2(u, y, Z_1, Z_2)\end{aligned} \tag{14}$$

$$\dot{\epsilon}_3 = -\varpi S_3^{-1} \Lambda^T C_1^T C_1 \epsilon_1' - B_1(Z_2) C_2 \epsilon_2 - B_2(Z_2) C_1 \epsilon_1'. \tag{15}$$

Following the same idea as in Zhang (2002), and applying the transformation $\epsilon_1 = \epsilon'_1 - \Lambda \epsilon_3$, it yields

$$\dot{\epsilon}_1 = \dot{\epsilon}_1' - \Lambda \dot{\epsilon}_3 - \dot{\Lambda} \epsilon_3. \tag{16}$$

By substituting (16) into (13)–(15), the estimation error dynamics are given by

$$\begin{aligned}\dot{\epsilon}_1 = {} & [A_1(Z_2, y) - \Gamma S_1^{-1} C_1^T C_1 + B_{21}]\epsilon_1 \\ & + g_1(u, y, X_2, X_1) + \Delta g_1(u, y, X_2, X_1) \\ & - g_1(u, y, Z_2, Z_1) + (B_{12} - K')\epsilon_2 + B_{22}\epsilon_3 \\ & + [A_1(X_2, y) + \Delta A_1(X_2, y) - A_1(Z_2, y)]X_1 \\ \dot{\epsilon}_2 = {} & [A_2(Z_1) - S_2^{-1} C_2^T C_2]\epsilon_2 + [A_2(X_1) \\ & + \Delta A_2(X_1) - A_2(Z_1)]X_2 g_2(u, y, X_1, X_2) \\ & - g_2(u, y, Z_1, Z_2) + \Delta g_2(u, y, X_1, X_2) \\ \dot{\epsilon}_3 = {} & -[\varpi S_3^{-1} \Lambda^T C_1^T C_1 \Lambda + B_2']\epsilon_3 \\ & - [\varpi S_3^{-1} \Lambda^T C_1^T C_1 + B_2'']\epsilon_1 - B_1' \epsilon_2\end{aligned} \tag{17}$$

with $B_{21} = \Lambda B_2(Z_2) C_1$, $B_{12} = \Lambda B_1(Z_2) C_2$, $B_{22} = \Lambda B_2 \times (Z_2) C_1 \Lambda$, $B_2{}' = B_2(Z_2) C_1 \Lambda$, $B_2{}'' = B_2(Z_2) C_1$, $B_1{}' = B_1 \times (Z_2) C_2$, $K' = K C_2^T C_2$. Since (u, X_2) and (u, X_1) are regular persistent inputs for subsystems (10) and (11), respectively, and from Lemma 3.6, then there exist $t_0 \ge 0$ and real numbers $\eta_{S_i}^{\max} > 0$, $\eta_{S_i}^{\min} > 0$ which are independent of θ_i such that $V(t, \epsilon_i) = \epsilon_i^T S_i \epsilon_i$ $(1 \le i \le 3)$ (Besançon and Hammouri 1996)

$$\forall t \ge t_0 \quad \eta_{S_i}^{\min} \|\epsilon_i\|^2 \le V(t, \epsilon_i) \le \eta_{S_i}^{\max} \|\epsilon_i\|^2. \tag{18}$$

Theorem 4.3: *Consider the extended* IM *dynamic model represented by* (3) *and* (4). *Systems* (5) *and* (6) *is an adaptive observer for systems* (3) *and* (4). *Furthermore, the strongly uniformly practical stability of estimation error dynamics* (17) *is established.* □

Proof of Theorem 4.3: A Lyapunov function candidate is considered as $V_0 = V_1 + V_2 + V_3$, where $V_1 = \epsilon_1^T S_1 \epsilon_1$, $V_2 = \epsilon_2^T S_2 \epsilon_2$ and $V_3 = \epsilon_3^T S_3 \epsilon_3$. Taking the time derivative of V_0 and using (5), (6) and (17), we have

$$\begin{aligned}\dot{V}_0 = {} & \epsilon_1^T\{-\theta_1 S_1 - (2 S_1 \Gamma S_1^{-1} - 1) C_1^T C_1 \\ & + 2 S_1 B_{21}\}\epsilon_1 + 2\epsilon_1^T S_1 \{A_1(X_2, y) \\ & - A_1(Z_2, y) + \Delta A_1(X_2, y)\} X_1 \\ & + 2\epsilon_1^T S_1 \{g_1(u, y, X_2, X_1) - g_1(u, y, Z_2, Z_1) \\ & + \Delta g_1(u, y, X_2, X_1)\} + \epsilon_3^T[-\theta_3 S_3 - (2\varpi - 1)\Lambda^T C_1^T C_1 \Lambda \\ & - 2 S_3 B_2']\epsilon_3 + \epsilon_2^T\{-\theta_2 S_2 - C_2^T C_2\}\epsilon_2 \\ & + 2\epsilon_2^T S_2 \{A_2(X_1) - A_2(Z_1) + \Delta A_2(X_1)\} X_2 \\ & + 2\epsilon_2^T S_2 \{g_2(u, y, X_1, X_2) - g_2(u, y, Z_1, Z_2) \\ & + \Delta g_2(u, y, X_1, X_2)\} \\ & + 2\epsilon_1^T S_1 (B_{12} - K')\epsilon_2 + 2\epsilon_1^T S_1 B_{22} \epsilon_3 \\ & - 2\epsilon_3^T (B_2'' + \varpi \Lambda^T C_1^T C_1)\epsilon_1 - 2\epsilon_3^T S_3 B_1' \epsilon_2.\end{aligned} \tag{19}$$

According to Lemma 3.6 and tacking the initial conditions of the IM drive and the observer in the physical operation domain $\mathcal{D}$, the following inequalities hold:

$$\begin{aligned}& \|S_1\| \le k_1, \ \|S_2\| \le k_5, \ \|X_1\| \le k_3, \ \|X_2\| \le k_7 \\ & \|\{g_1(u, y, X_2, X_1) - g_1(u, y, Z_2, Z_1)\}\| \le k_4 \|\epsilon_2\| + k_{16} \|\epsilon_1\| \\ & \|\{A_1(X_2, y) - A_1(Z_2, y)\}\| \le k_2 \|\epsilon_2\| \\ & \|\{A_2(X_1) - A_2(Z_1)\}\| \le k_6 \|\epsilon_1\| + k_{20} \|\epsilon_3\| \\ & \|\{g_2(u, y, X_1, X_2) - g_2(u, y, Z_1, Z_2)\}\| \le k_8 \|\epsilon_1\| + k_{17} \|\epsilon_2\| \\ & \|B_1'\| \le k_9, \ \|B_2'\| \le k_{10}, \ \|B_2''\| \le k_{18}, \ \|B_{12}\| \le k_{11}, \\ & \|B_{21}\| \le k_{12}, \ \|B_{22}\| \le k_{13}, \ \|K'\| \le k_{14} \\ & \|\Lambda^T C_1^T C_1\| \le k_{19}, \ \|S_3\| \le k_{15}.\end{aligned} \tag{20}$$

Substituting Equation (20) into (19), from Assumption 3 and by regrouping with respect to $\|\epsilon_1\|$, $\|\epsilon_2\|$ and $\|\epsilon_3\|$, the time derivative of V_0 (19) can be rewritten as follows:

$$\begin{aligned}\dot{V}_0 \le {} & -(\theta_1 - 2k_{12} - 2k_1 k_{16})\epsilon_1^T S_1 \epsilon_1 \\ & - (\theta_2 - 2k_5 k_{17})\epsilon_2^T S_2 \epsilon_2 - (\theta_3 + 2k_{10})\epsilon_3^T S_3 \epsilon_3 \\ & + 2(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)\|\epsilon_1\|\|\epsilon_2\| \\ & + 2\mu_9 \|\epsilon_2\|\|\epsilon_3\| + 2\mu_8 \|\epsilon_1\|\|\epsilon_3\| \\ & + \mu_6 \|\epsilon_1\| + \mu_7 \|\epsilon_2\|\end{aligned} \tag{21}$$

where $\mu_1 = k_1k_2k_3$, $\mu_2 = k_1k_4$, $\mu_3 = k_5k_6k_7$, $\mu_4 = k_5k_8$, $\mu_5 = k_1(k_{11} - k_{14})$, $\mu_6 = 2(k_1k_3\rho_1 + k_1\rho_3)$, $\mu_7 = 2(k_5k_7\rho_2 + k_5\rho_4)$, $\mu_8 = k_1k_{13} - (\varpi k_{19} + k_{18})$, $\mu_9 = -k_{15}k_9 + k_5k_{20}k_7$.

Inequality (21) can be rewritten in terms of V_1, V_2 and V_3 as follows:

$$\begin{aligned}\dot{V}_0 \le &-(\theta_1 - 2k_{12} - 2k_1k_{16})V_1 - (\theta_2 - 2k_5k_{17})V_2\\ &- (\theta_3 + 2k_{10})V_3 + 2\tilde{\mu}\sqrt{V_1}\sqrt{V_2} + \tilde{\mu}_7\sqrt{V_2}\\ &+ 2\tilde{\mu}_9\sqrt{V_2}\sqrt{V_3} + \tilde{\mu}_6\sqrt{V_1} + 2\tilde{\mu}_8\sqrt{V_1}\sqrt{V_3} \end{aligned}\tag{22}$$

where $\tilde{\mu} = \sum_{j=0}^{5}\tilde{\mu}_j$, $\tilde{\mu}_j = \frac{\mu_j}{\sqrt{\eta_{S_1}^{\min}}\sqrt{\eta_{S_2}^{\min}}}$, $j = 1, \ldots, 5$; $\tilde{\mu}_8 = \frac{\mu_8}{\sqrt{\eta_{S_1}^{\min}}\sqrt{\eta_{S_3}^{\min}}}$, $\tilde{\mu}_9 = \frac{\mu_9}{\sqrt{\eta_{S_2}^{\min}}\sqrt{\eta_{S_3}^{\min}}}$, $\tilde{\mu}_6 = \frac{\mu_6}{\sqrt{\eta_{S_1}^{\min}}}$, $\tilde{\mu}_7 = \frac{\mu_7}{\sqrt{\eta_{S_2}^{\min}}}$.
Using the following inequalities:

$$\begin{aligned}\sqrt{V_1}\sqrt{V_2} &\le \frac{\varphi_1}{2}V_1 + \frac{1}{2\varphi_1}V_2\\ \sqrt{V_1}\sqrt{V_3} &\le \frac{\varphi_2}{2}V_1 + \frac{1}{2\varphi_2}V_3\\ \sqrt{V_2}\sqrt{V_3} &\le \frac{\varphi_3}{2}V_2 + \frac{1}{2\varphi_3}V_3 \quad \forall\varphi_i(i = 1, 2, 3) \in]0, 1[, \end{aligned}\tag{23}$$

by substituting (23) into (22), we obtain

$$\begin{aligned}\dot{V}_0 \le &-(\theta_1 - 2k_{12} - 2k_1k_{16} - \tilde{\mu}\varphi_1 - \tilde{\mu}_8\varphi_2)V_1\\ &- (\theta_2 - 2k_5k_{17} - \frac{\tilde{\mu}}{\varphi_2} - \tilde{\mu}_9\varphi_3)V_2\\ &- (\theta_3 + 2k_{10} - \frac{\tilde{\mu}_8}{\varphi_2} - \frac{\tilde{\mu}_9}{\varphi_3})V_3\\ &+ \tilde{\mu}_6\|\epsilon_1\| + \tilde{\mu}_7\|\epsilon_2\|, \end{aligned}\tag{24}$$

and consequently, we have

$$\begin{aligned}\dot{V}_0 &\le -\delta(V_1 + V_2 + V_3) + \mu(\sqrt{V_1} + \sqrt{V_2})\\ &\le -\delta V_0 + \mu\psi\sqrt{V_0}, \end{aligned}\tag{25}$$

where $\delta = \min(\delta_1, \delta_2, \delta_3)$, $\mu = \max(\tilde{\mu}_6, \tilde{\mu}_7)$, where $\delta_1 = \theta_1 - 2k_{12} - 2k_1k_{16} - \tilde{\mu}\varphi_1 - \tilde{\mu}_8\varphi_2 > 0$, $\delta_2 = \theta_2 - 2k_5k_{17} - \frac{\tilde{\mu}}{\varphi_2} - \tilde{\mu}_9\varphi_3 > 0$, $\delta_3 = \theta_3 + 2k_{10} - \frac{\tilde{\mu}_8}{\varphi_2} - \frac{\tilde{\mu}_9}{\varphi_3} > 0$, and $\psi > 0$, such that $\psi\sqrt{V_1 + V_2 + V_3} > \sqrt{V_1} + \sqrt{V_2}$. So that

$$\begin{aligned}\theta_1 &> 2k_{12} + 2k_1k_{16} + \tilde{\mu}\varphi_1 + \tilde{\mu}_8\varphi_2\\ \theta_2 &> 2k_5k_{17} + \frac{\tilde{\mu}}{\varphi_2} + \tilde{\mu}_9\varphi_3\\ \theta_3 &> \frac{\tilde{\mu}_8}{\varphi_2} + \frac{\tilde{\mu}_9}{\varphi_3} - 2k_{10}. \end{aligned}\tag{26}$$

Next, consider the following change of variable $v = 2\sqrt{V_0}$, the time derivative of v is given by

$$\dot{v} \le -\delta v + \psi\mu. \tag{27}$$

From Theorem 4.1 we have $\wp(t, l) = -\delta l + \psi\mu$, therefore (9) can be expressed as

$$\dot{l} = -\delta l + \psi\mu, \quad l(t_0) = l_0 \ge 0 \tag{28}$$

and its solution is given as

$$l(t) = l(t_0)e^{-\delta(t-t_0)} + r\cdot(1 - e^{-\delta(t-t_0)}) \tag{29}$$

where $r = \frac{\psi\mu}{\delta}$ depends on parameters θ_i $(i = 1, 2, 3)$.

Now, in order to prove the strongly uniformly practical stability (refer to Corollary 4.2) of (28), first let us prove the uniform practical stability. Suppose that $l(t_0) \le \hbar_1$. Then, from (29) we have:

$$\begin{aligned}l(t) &\le l(t_0) + r\\ &\le \hbar_1 + r \le \hbar_2 \end{aligned}\tag{30}$$

so that $l(t_0) \le \hbar_1$ implies $l(t) \le \hbar_2$, $\forall t \ge t_0$. According to (PS1), (28) is uniformly practically stable.

Next, let us prove the uniformly quasi-practical stability of (28). Suppose that there exist $\hbar_1 > 0$, $\Im > 0$, $T > 0$, $l(t_0) \le \hbar_1$ and $t \ge t_0 + T$. Equation (29) verifies the following inequality:

$$\begin{aligned}l(t) &\le l(t_0)e^{-\delta T} + r\\ &\le \hbar_1 e^{-\delta T} + r \le \Im \end{aligned}\tag{31}$$

so that $l(t_0) \le \hbar_1$ implies $l(t) \le \Im$, $\forall t \ge t_0 + T$. According to (PS2), (28) is uniformly practically quasi-stable.

Then, according to definition (PS3), (28) is strongly uniformly practically stable.

In order to prove the strong uniform practical stability of (17), we check all the conditions of Theorem 4.1. It is clear that from (30) and (31), $\hbar_1 < \hbar_2$, $\Im < \hbar_2$, then condition (i) of Theorem 4.1 is satisfied.

Now by using (18), we have $\eta^{\min}\|e\|^2 \le V_0(t, e) \le \eta^{\max}\ \|e\|^2$, $V_0(t, e)$ is a Lyapunov function, locally Lipschitz in e, where $\eta^{\min} = \min\{\eta_{S_i}^{\min}, i = 1, 2, 3\}$ and $\eta^{\max} = \max\{\eta_{S_i}^{\max}, i = 1, 2, 3\}$. Taking $d_1(\|e\|) = \eta^{\min}\|e\|^2$, $d_2(\|e\|) = \eta^{\max}\|e\|^2$. Next, for $(t, e) \in \mathbb{R}_+ \times B_{\hbar_2}$, $d_1(\|e\|) \le V_0(t, e) \le d_2(\|e\|)$ and from (25) $\wp(t, V_0(t, e)) = -\delta V_0 + \mu\psi\sqrt{V_0}$. Then, the conditions (ii) and (iii) of Theorem 4.1 are verified.

Next, we prove condition (iv) of Theorem 4.1. On one hand $v(t_0) \le \hbar_1$ (because $l(t_0) \le \hbar_1$) implies $v(t) \le \hbar_2$ (because $l(t) \le \hbar_2$), $\forall t \ge t_0$. Moreover, $V_0(t, e) = \frac{1}{4}v(t)^2$. Then, it follows that $v(t_0) \le \hbar_1$, implies $\eta^{\min}\|e_0\|^2 < \frac{1}{4}\hbar_1^2$. Hence, $\|e_0\| < \frac{1}{2\sqrt{\eta^{\min}}}\hbar_1$.

On the other hand, $\frac{1}{4}v(t)^2 = V_0(t, e) = \eta^{\max}\|e(t)\|^2 < \frac{1}{4}\hbar_2^2$. Hence, $\|e(t)\| < \frac{1}{2\sqrt{\eta^{\max}}}\hbar_2$. This proves the uniform practical stability of (17). Consequently $0 < \frac{1}{2\sqrt{\eta^{\min}}}\hbar_1 < \frac{1}{2\sqrt{\eta^{\max}}}\hbar_2$. Writing the above inequality in the following form, we have $\eta^{\max}\hbar_1^2 < \eta^{\min}\hbar_2^2$, that implies $d_2(\hbar_1) < d_1(\hbar_2)$. Finally, all conditions of Theorem 4.1 hold. Then, this implies the strongly uniformly practical stability of (17).

Remark 4.4:

(1) Inequality (26) depends on the Lipschitz constants. From Lipschitz constants, we can calculate the minimum value of θ_i ($i = 1, 2, 3$). Then, we tune θ_i in order to precisely adjust the time convergence of the observer.
(2) In inequality (25), μ depends on parametric uncertainties (see μ_6 and μ_7). If IM parameters are exactly known $\mu = 0$ otherwise (under uncertainties) $\mu \neq 0$. Then the precision of the observer depends on μ, ψ and δ. The radius of the ball in the practical stability proof is $r = \frac{\psi\mu}{\delta}$. This radius can be adjusted by tuning $\delta(\theta_i)$.

5. Experimental results

The proposed observer algorithm has been tested using a 1.5 kW IM, whose data are reported in Tables 1 and 2.

The experimental set-up is equipped with:

(1) Three phases inverter operated with a symmetrical PWM technique with 5 kHz switching frequency.
(2) A permanent magnet synchronous motor controlled by industrial drive and used to provide a desired speed.
(3) A custom floating-point digital signal processor dSPACE (DS1103) board, and its interface. The dSPACE board performs data acquisition (two stator currents, DC-link voltage, load torque and rotor speed, by means of a 512 ppr incremental encoder, and only for monitoring purposes), computes the control algorithm and generates the PWM signals for the inverter actuation.

Table 1. Motor parameters values of the set-up.

Nominal rate power	1.5 kW
Nominal angular speed	1430 rpm
Number of pole pairs	2
Nominal voltage	220 V
Nominal current	7.5 A

Table 2. Motor identified parameters.

R_s	1.633 Ω	M_{sr}	0.099 H
R_r	0.93 Ω	J	0.0111 Nm s rad^{-1}
L_s	0.142 H	f_v	0.0018 Nm s rad^{-1}
L_r	0.076 H		

The algorithms implemented in the dSPACE board have a total time computational cost of 100 μs. The experimental sampling time T is equal to 200 μs. The parameters were chosen as follows: $\alpha = 0.01$, $k = 0.012$, $k_{c1} = 0.01$, $k_{c2} = 0.01$, $\varpi = 5$, $\theta_1 = 2000$, $\theta_2 = 3400$ and $\theta_3 = 2$ to satisfy convergence conditions.

The identified parameters of the IM have previously been obtained off-line and they are assumed to be close of the real values of the IM, it is clear that they are not exactly the real values. However, in the sequel these identified parameters are used to represent the so-called 'nominal system' and they will be used to start the test on the performance of the proposed observer.

The experimental results for 'the nominal case' (no uncertainties) are shown in Figure 1. These figures show that the estimated speed (Figure 1(a)) and the estimated load torque (Figure 1(d)) converge to their actual values (Figure 1(b)) and (Figure 1(c)), respectively. From this figure, we can see that the good performance of the observer under 'nominal' conditions. In this test, the initial value of the stator resistance is set equal to the identified value, and it is shown (Figure 1(e)) its convergence to a steady state value which is almost independent of the operating condition (observable and unobservable area). The convergence of the resistance estimator also reduces the speed observation error. From this figure, it follows that the estimated stator resistance rated value is constant. However, the behaviour of the observer is affected near and under conditions of unobservability. Comparing to the observer given in Ghanes et al. (2006), we noted that the estimated speed and load torque are not unstable near and under conditions of unobservability. However under these conditions, the asymptotically convergence of the errors dynamics is not ensured due to the inputs are not persistent in this zone. This is why we have introduced the practical stability.

The robustness of the observer is tested by introducing +50% variation on rotor resistance value used in the observer parameters (Figure 2). This figure displays similar experimental results for the stator resistance nominal case under unobservability conditions. Comparing to 'nominal case' (Figure 1), it appears a static error between the estimated speed (Figure 2(b)) and measured speed (Figure 2(a)) when the motor is under conditions of observability.

A second robustness test is made now by introducing −50% variation on rotor resistance value. The experimental results are shown in Figure 3. For the speed and load torque estimation, the conclusion is the same as +50% variation case (Figure 2).

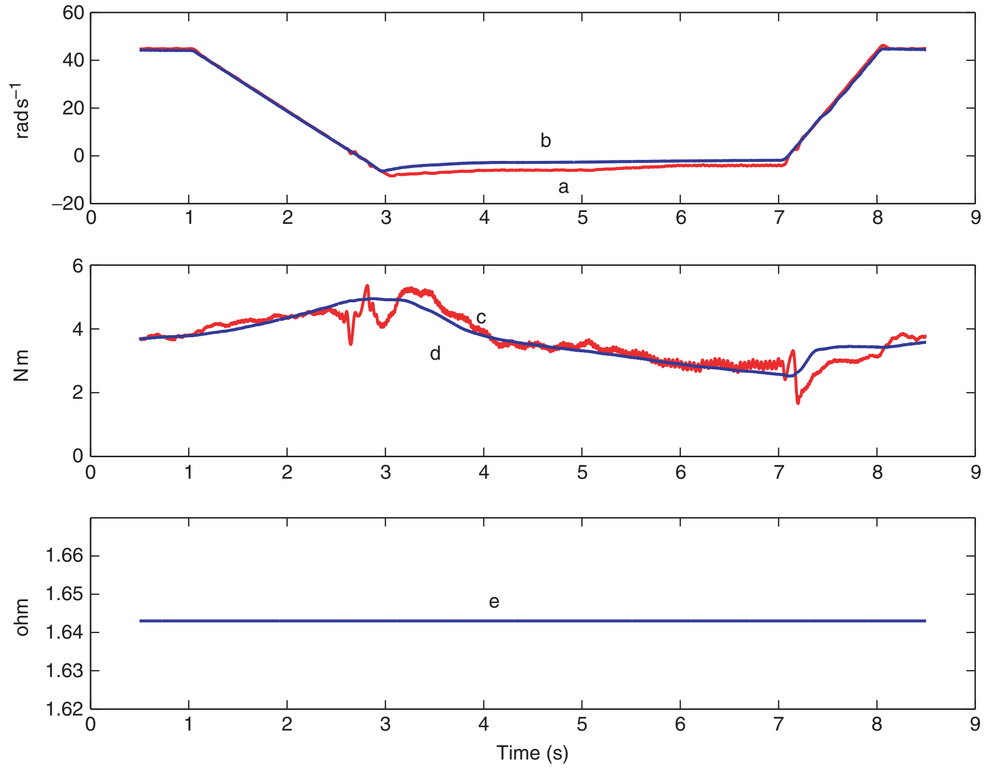

Figure 1. Nominal case. (a, c) measured speed (rad s^{-1}) and load torque. (Nm), (b, c, e) estimation speed (rad s^{-1}), load torque (Nm) and stator resistance (ohm).

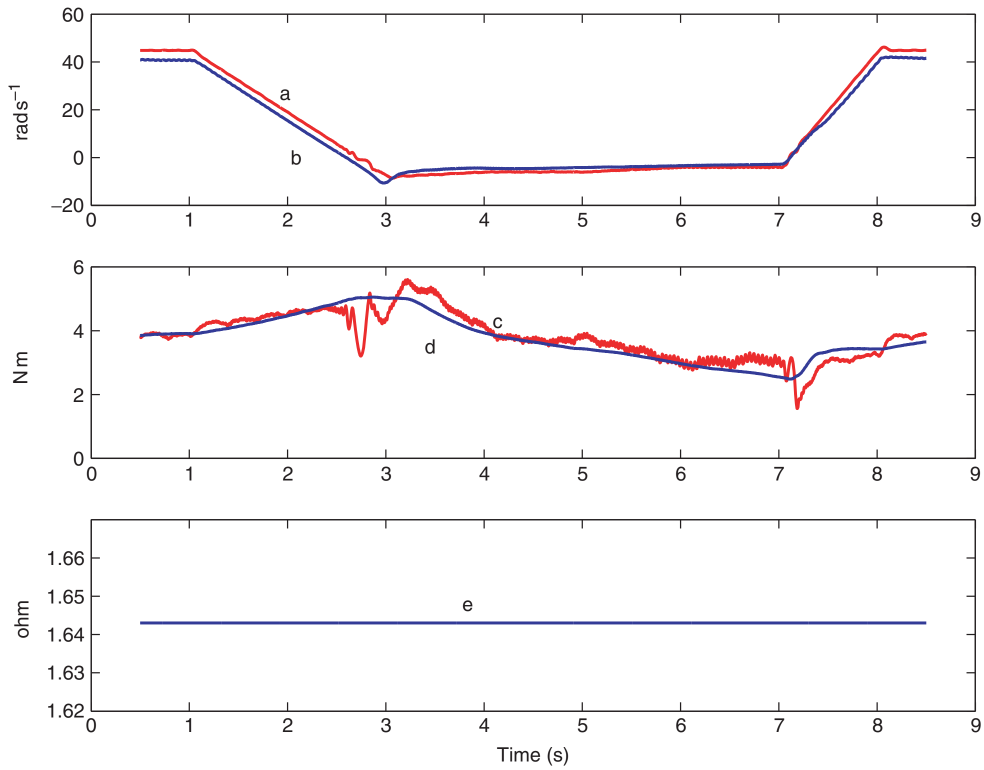

Figure 2. +50% rotor resistance variation (refer to Figure 1 for details of a–e).

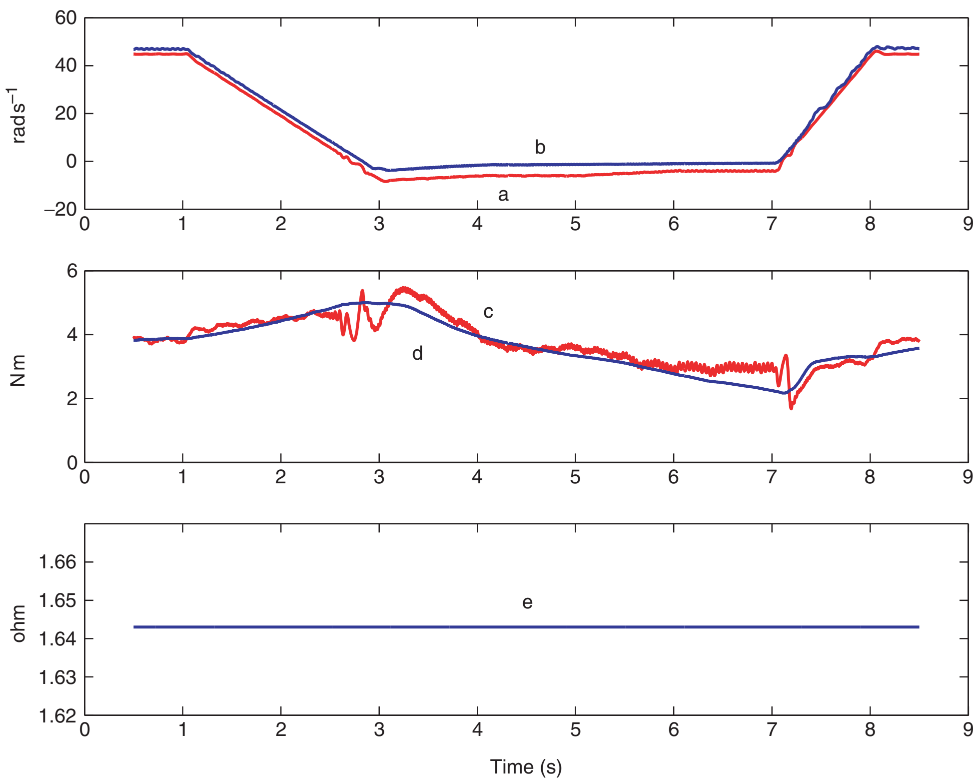

Figure 3. −50% rotor resistance variation (refer to Figure 1 for details of a–e).

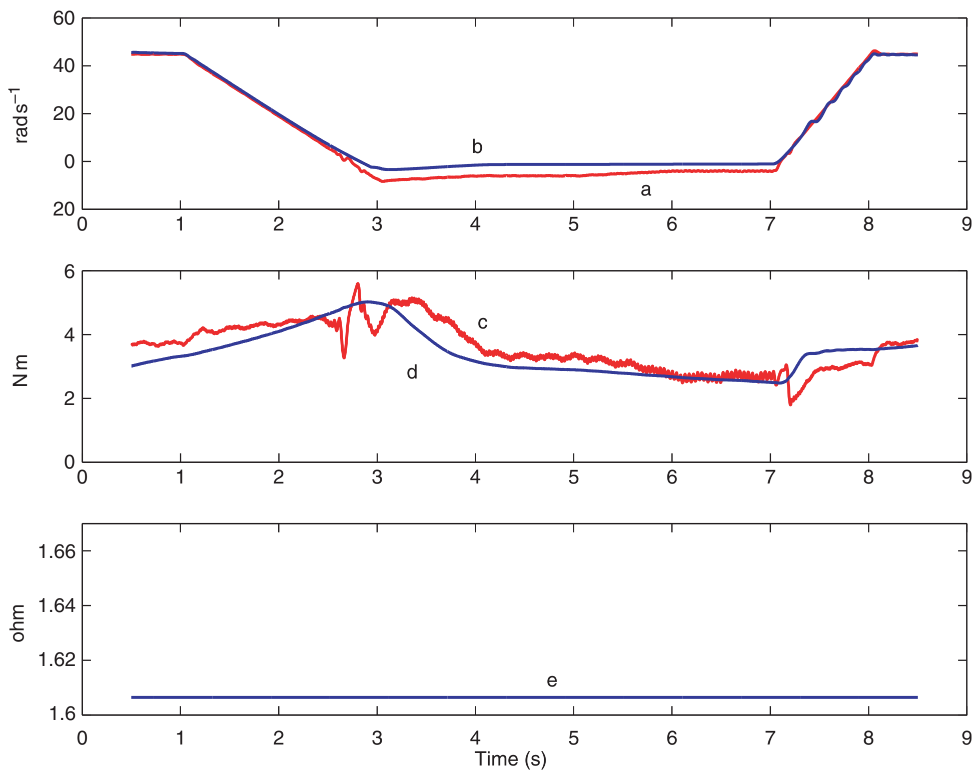

Figure 4. −20% stator resistance initial value variation (refer to Figure 1 for details of a–e).

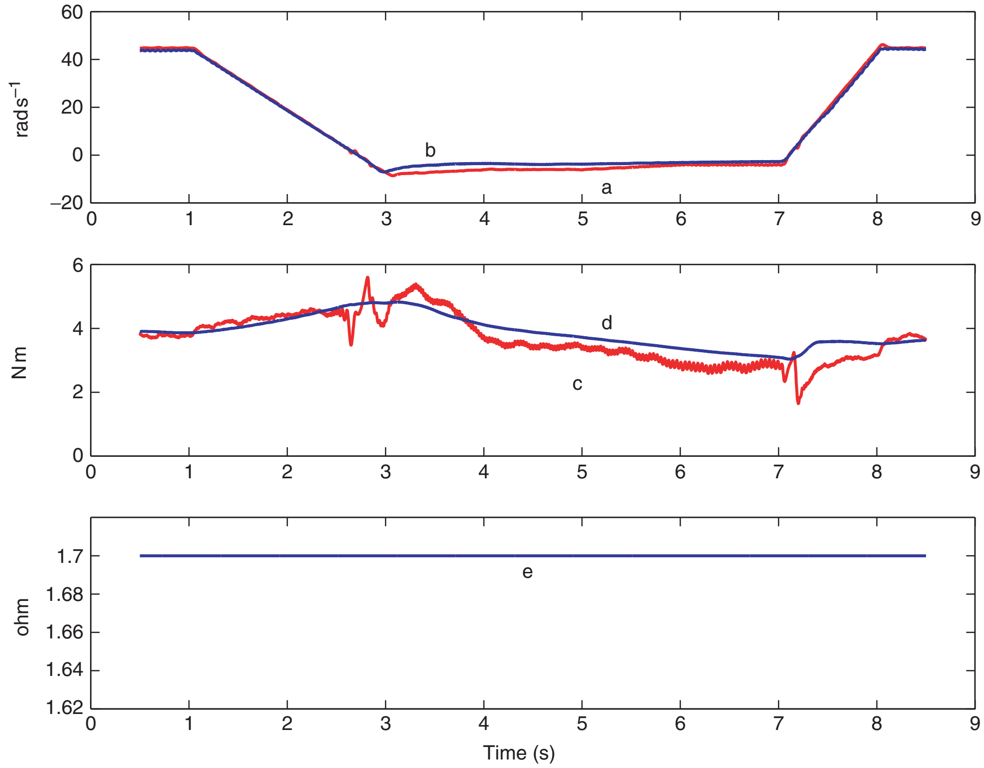

Figure 5. +20% stator resistance initial value variation (refer to Figure 1 for details of a–e).

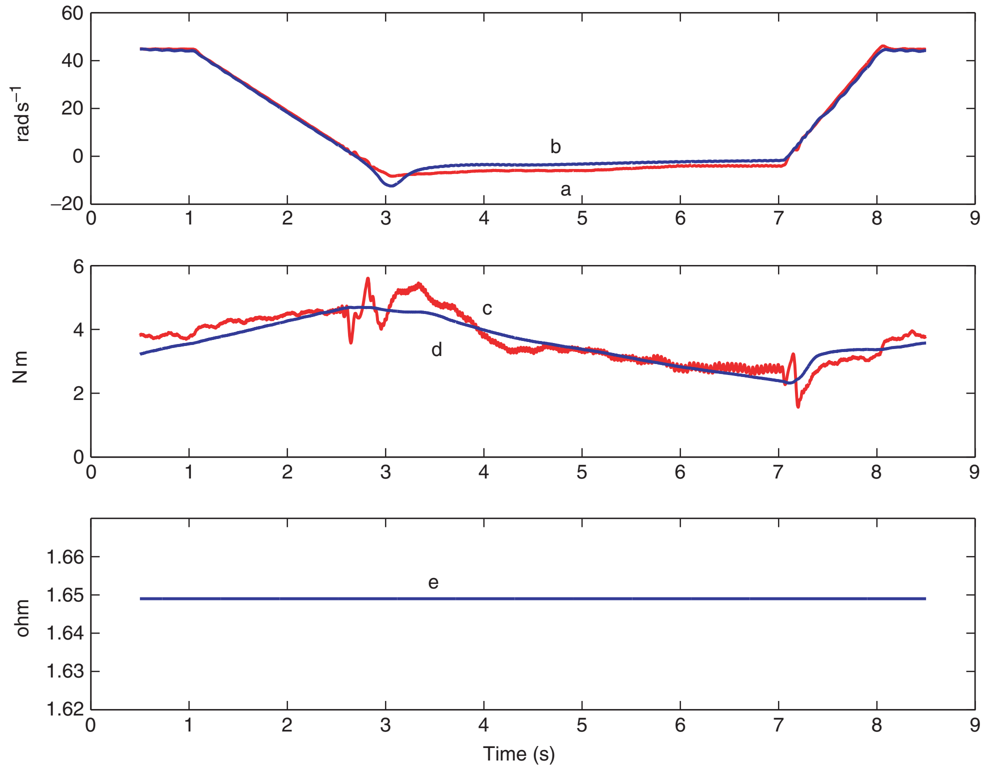

Figure 6. +20% rotor self-inductance variation (refer to Figure 1 for details of a–e).

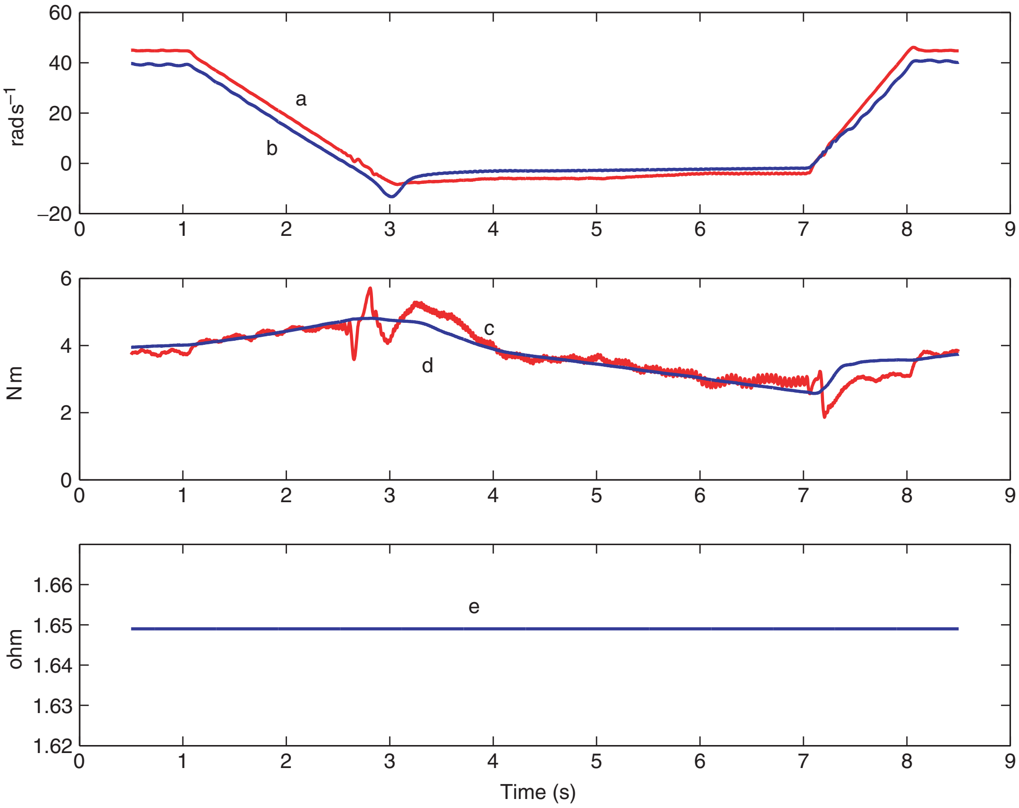

Figure 7. +20% stator self-inductance variation (refer to Figure 1 for details of a–e).

The aim of the next test is to show the good performances at high speed (under observability conditions) and low speed (under unobservability conditions i.e. low-excitation frequency), when the observer is initialised by a wrong value of the stator resistance $\hat{R}_s$ (nominal value $\pm$ 20%).

Figures 4 and 5 display an experimental result of this test with respectively -20% and $+20\%$ deviation for the stator resistance initial value. We conclude that the initialisation of the observer by the wrong value of stator resistance does not affect is performance. The estimated speed (Figures 4(a) and 5(a)) converges to the measured speed (Figures 4(b) and 5(b)). It is the same conclusion to the load torque; see respectively Figures 4(d) and 5(d) and Figures 4(c) and 5(c).

By comparing these results to the previous works (Ghanes et al. 2006; Traoré et al. 2006), we noted that the load torque is much better estimated. This is mainly due to the additional terms included in the proposed observer design and the better estimation of stator resistance rated value (Figures 4(e) and 5(e)).

A new test is made by introducing a variation of $+20\%$ on rotor self-inductance and on stator self-inductance values. The results of these tests are shown in Figures 6 and 7, respectively. By analysing these figures, it can be noted that stator self-inductance variation affects more the performance of the adaptive observer than rotor self-inductance variation.

6. Conclusion

This study has investigated the observer design for IM drive without mechanical sensors (speed sensor, load torque sensor). The major contributions of this study are:

(1) The design of an adaptive interconnected observer that estimates the rotor speed, the rotor fluxes, the load torque and the stator resistance (critical parameter at low speed).
(2) Based on Lyapunov theory, sufficient conditions have been given to prove the properties of the practical stability of the error estimation dynamics are satisfied even under or near unobservability conditions.
(3) The successful application of the observer scheme on experimental set-up with a significant sensorless observer benchmark.

The experimental results confirm that the observer can be applied to reconstruct the state at low frequencies (near and under conditions of turning unobservability) without turning off the observer gains at low frequencies. Moreover, the robustness of the observer is verified by introducing significant parameter variations tests.

Note

1. For details about choice of reference frames (in the current case, **dq**-frame), see Chiasson (1995).

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